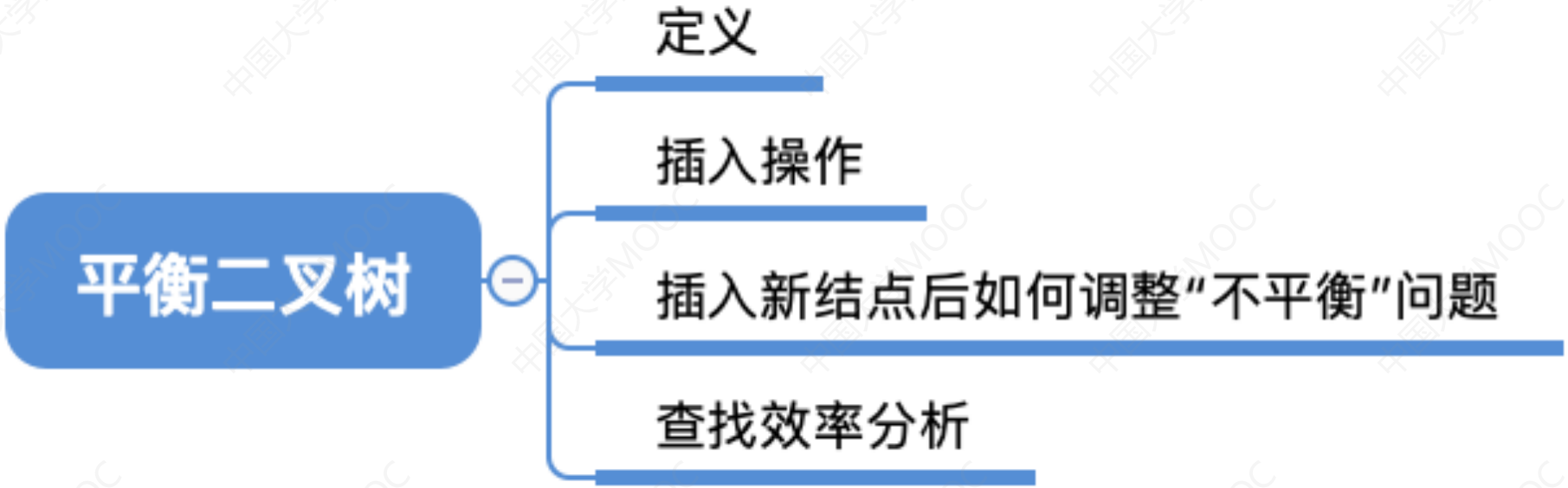
本节内容

平衡二叉树

(AVL)

知识总览



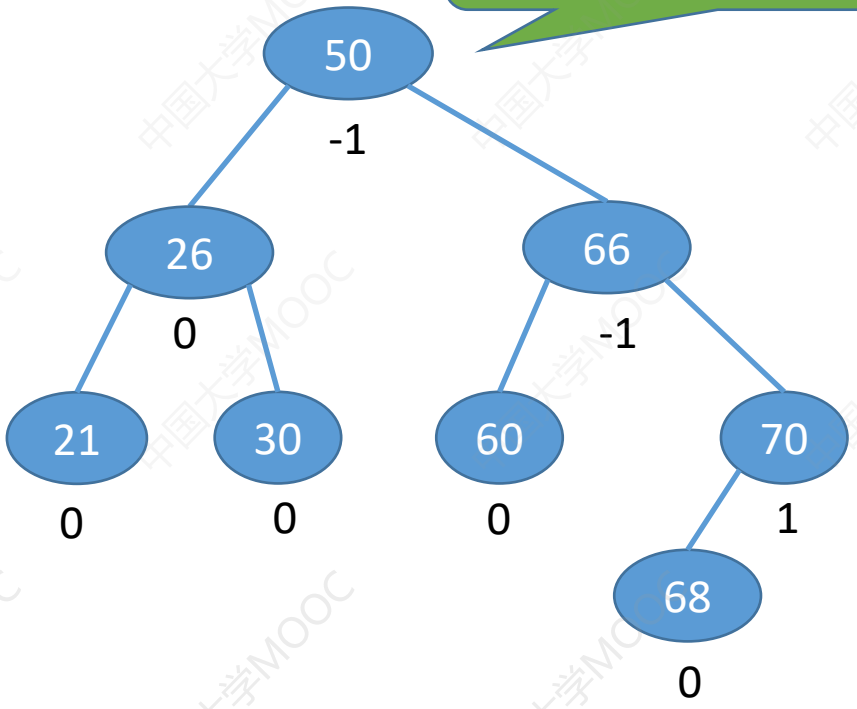
平衡二叉树的定义

G. M. Adelson-Velsky和
E. M. Landis

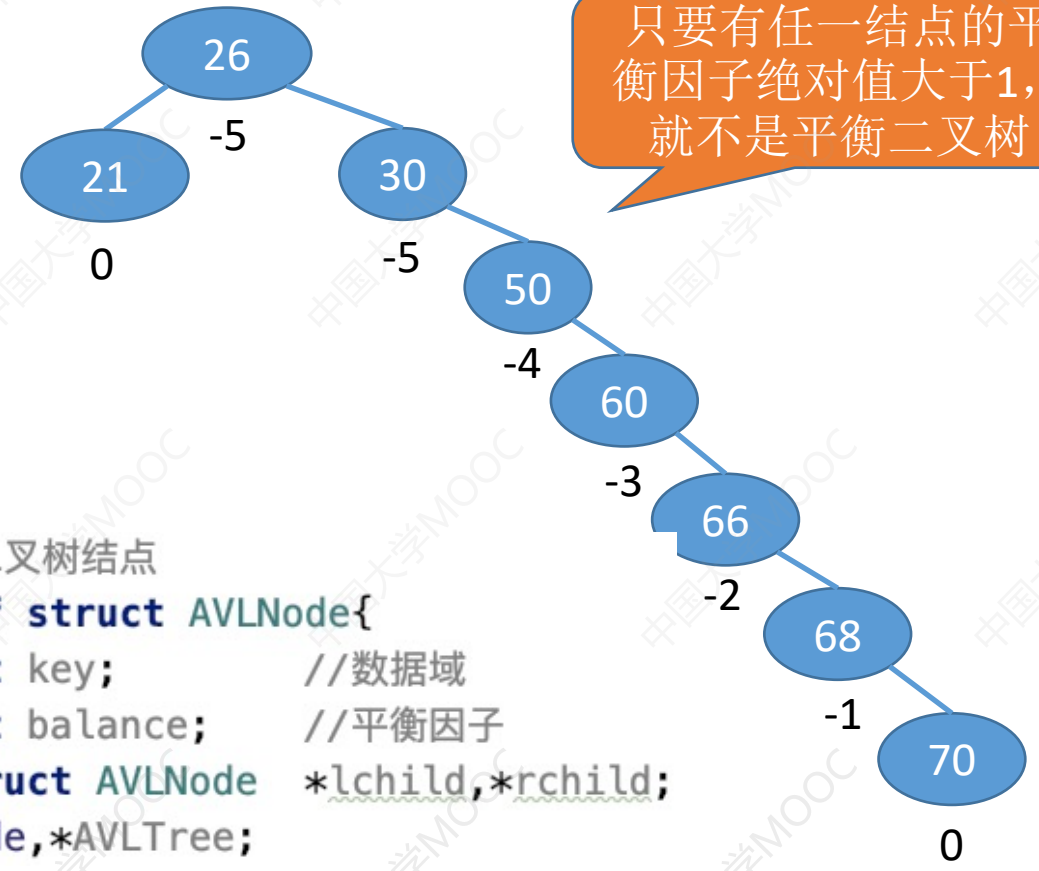
平衡二叉树（Balanced Binary Tree），简称平衡树（AVL树）——树上任一结点的左子树和右子树的高度之差不超过1。

结点的平衡因子=左子树高-右子树高。

平衡二叉树结点的平衡因子的值只可能是-1、0或1。



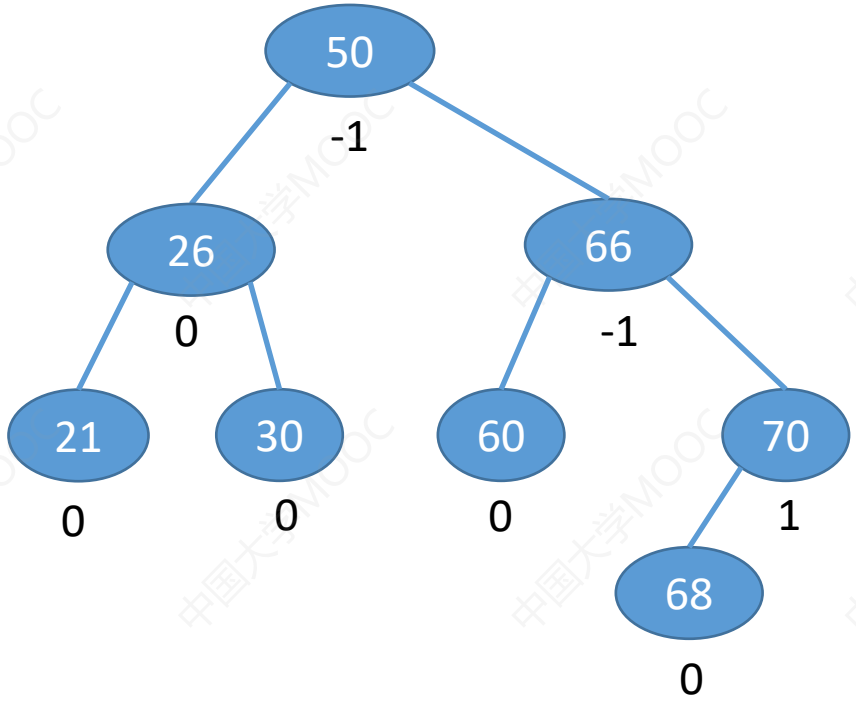
只要有任一结点的平衡因子绝对值大于1，就不是平衡二叉树



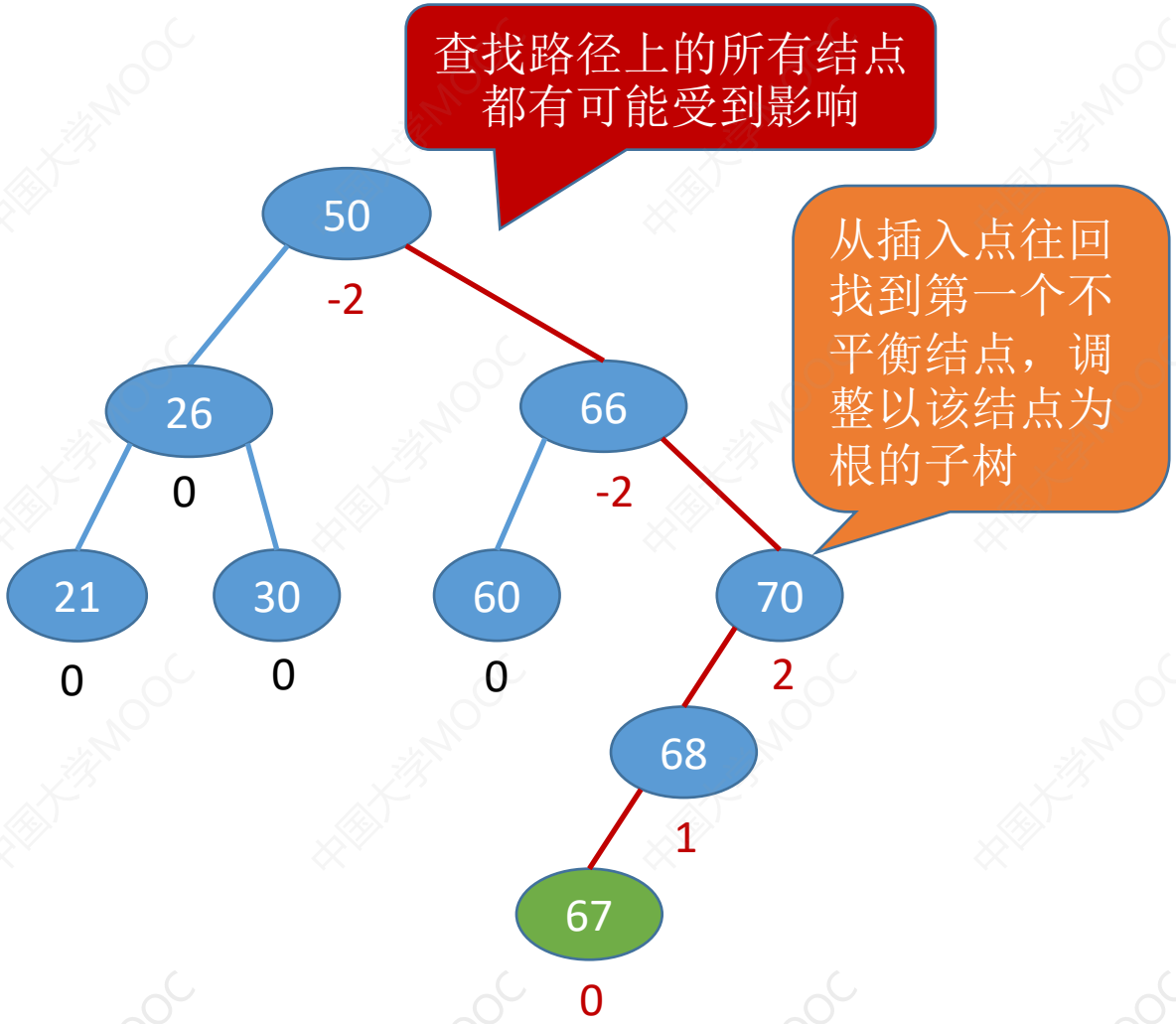

```
//平衡二叉树结点
typedef struct AVLNode{
    int key;          //数据域
    int balance;      //平衡因子
    struct AVLNode  *lchild,*rchild;
}AVLNode,*AVLTree;
```

平衡二叉树的插入

在二叉排序树中插入新结点后，如何保持平衡？

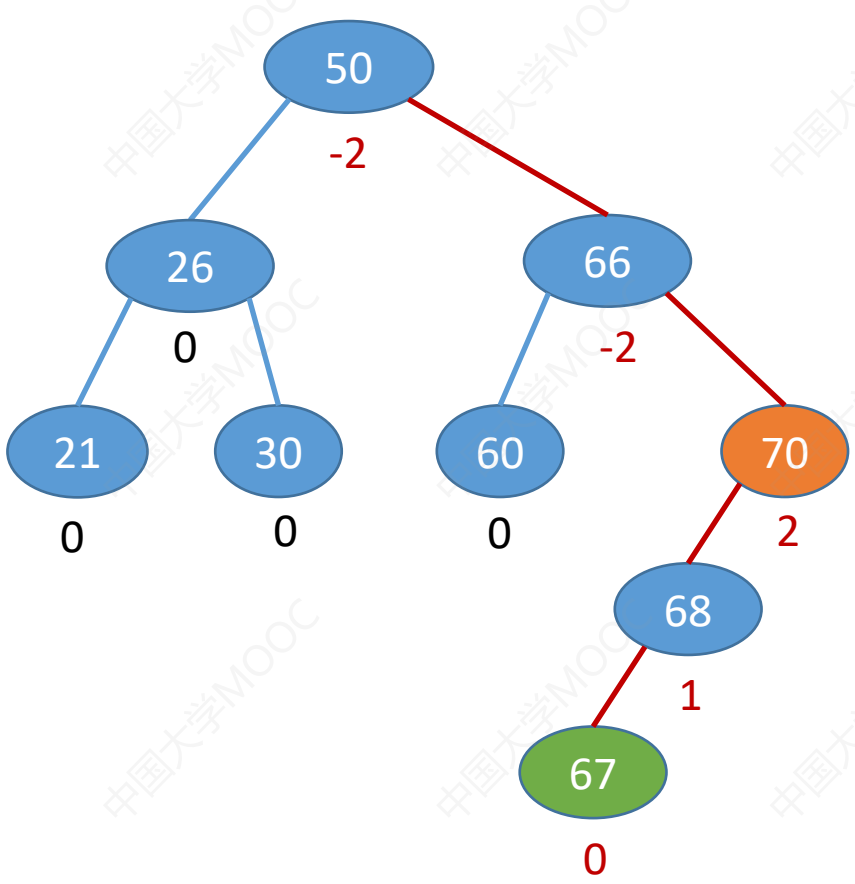


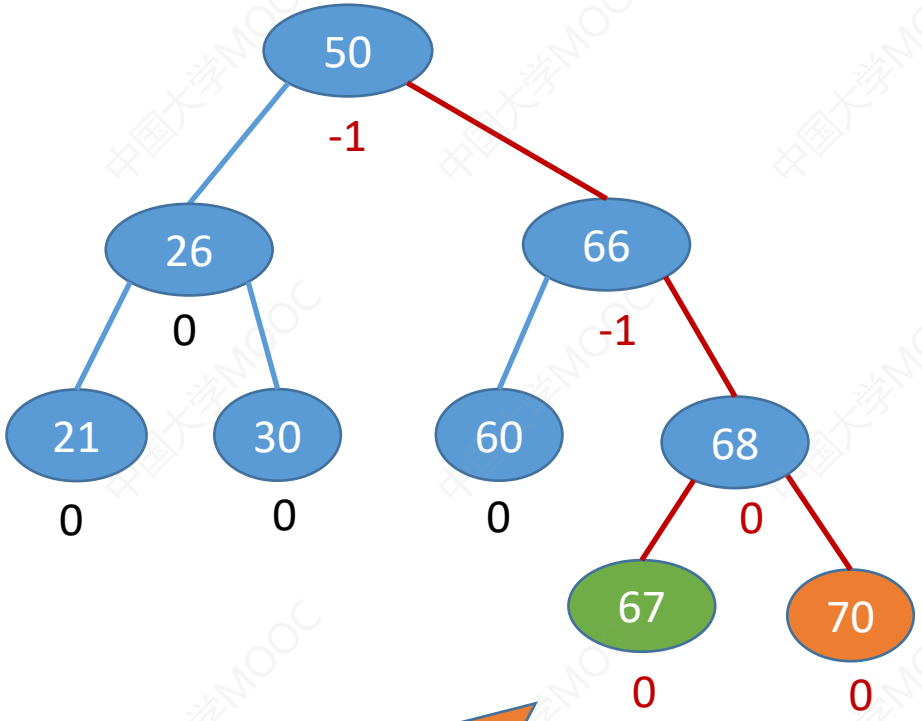
插入67



每次调整的对象都是“最小不平衡子树”

平衡二叉树的插入



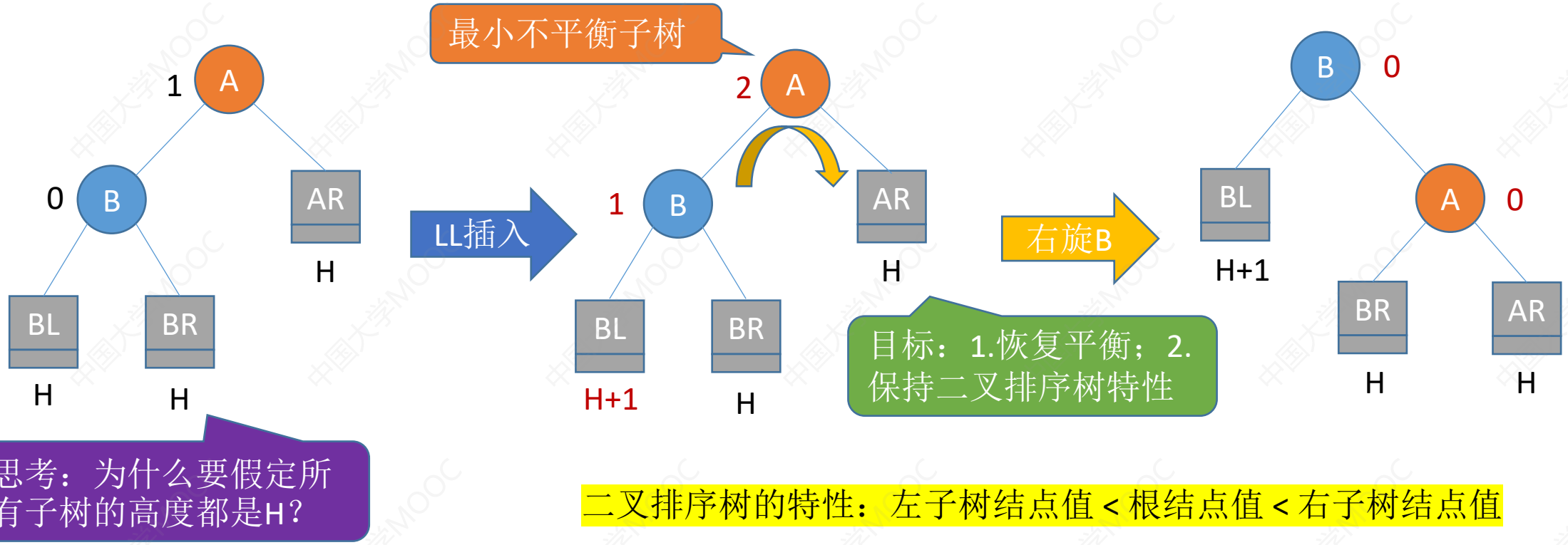


每次调整的对象都是“最小不平衡子树”

在插入操作中，只要将最小不平衡子树调整平衡，则其他祖先结点都会恢复平衡

调整最小不平衡子树

调整最小不平衡子树（LL）

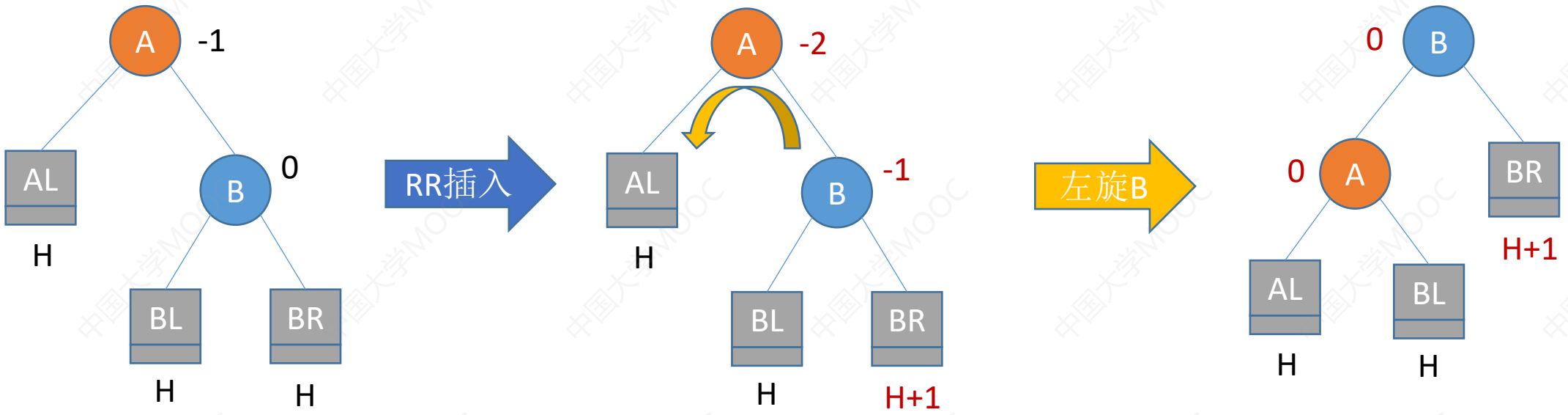


二叉排序树的特性：左子树结点值 < 根结点值 < 右子树结点值

BL<B<BR<A<AR

1）LL平衡旋转（右单旋转）。由于在结点*A*的左孩子（L）的左子树（L）上插入了新结点，*A*的平衡因子由1增至2，导致以*A*为根的子树失去平衡，需要一次向右的旋转操作。将*A*的左孩子*B*向右上旋转代替*A*成为根结点，将*A*结点向右下旋转成为*B*的右子树的根结点，而*B*的原右子树则作为*A*结点的左子树。

调整最小不平衡子树（RR）

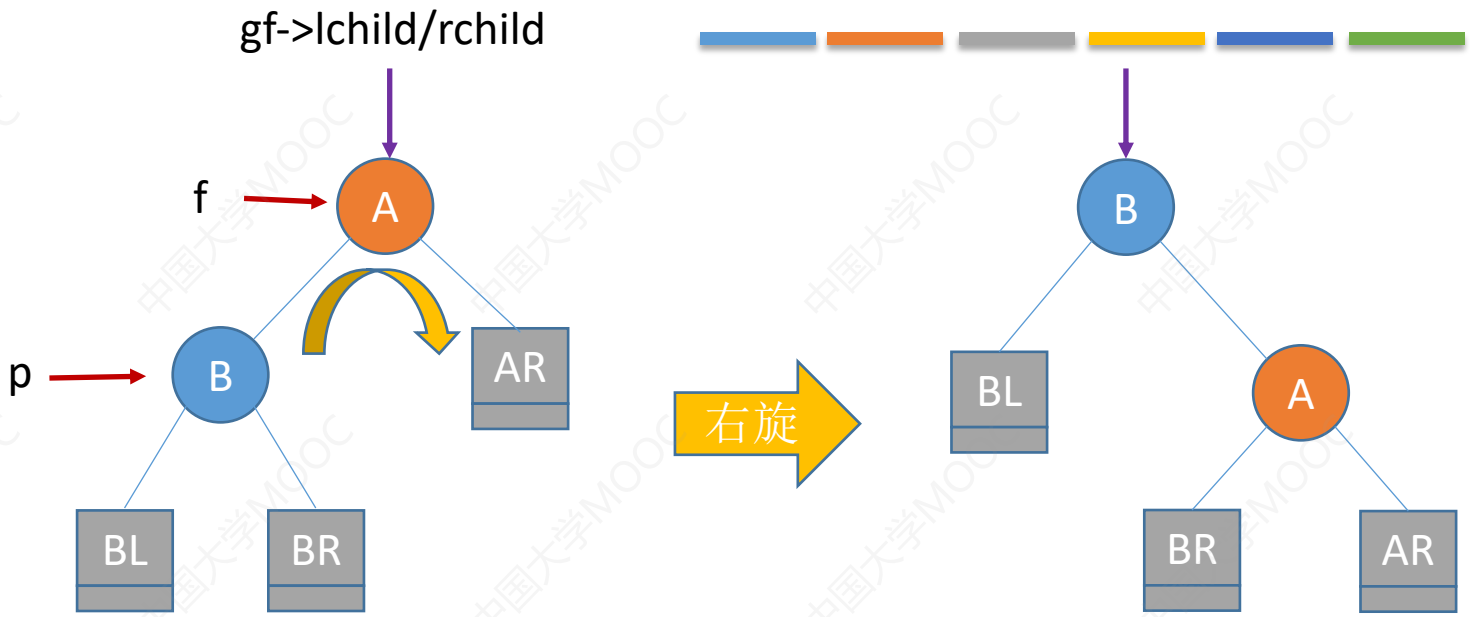


二叉排序树的特性：左子树结点值 < 根结点值 < 右子树结点值

AL<A<BL<B<BR

2）RR平衡旋转（左单旋转）。由于在结点*A*的右孩子（R）的右子树（R）上插入了新结点，*A*的平衡因子由-1减至-2，导致以*A*为根的子树失去平衡，需要一次向左的旋转操作。将*A*的右孩子*B*向左上旋转代替*A*成为根结点，将*A*结点向左下旋转成为*B*的左子树的根结点，而*B*的原左子树则作为*A*结点的右子树

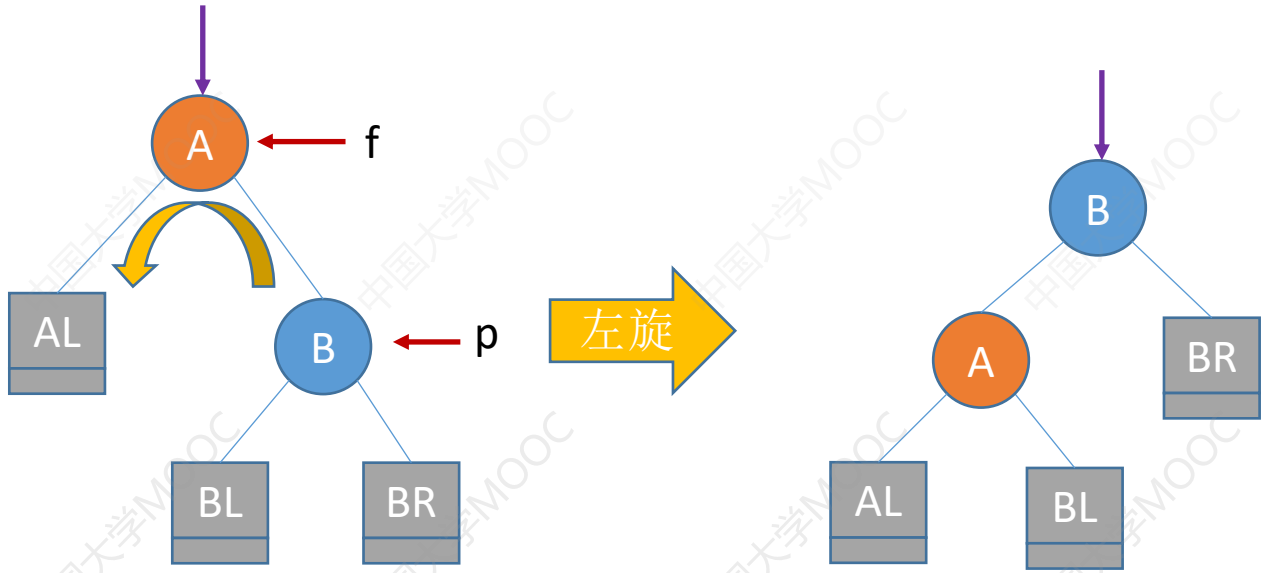
代码思路



实现 f 向右下旋转， p 向右上旋转：
其中 f是爹，p为左孩子，gf为f他爹
① f->lchild = p->rchild;
② p->rchild = f;
③ gf->lchild/rchild = p;

BL<B<BR<A<AR

左旋、右旋操作后可以保持二叉排序树的特性



实现 f 向左下旋转， p 向左上旋转：
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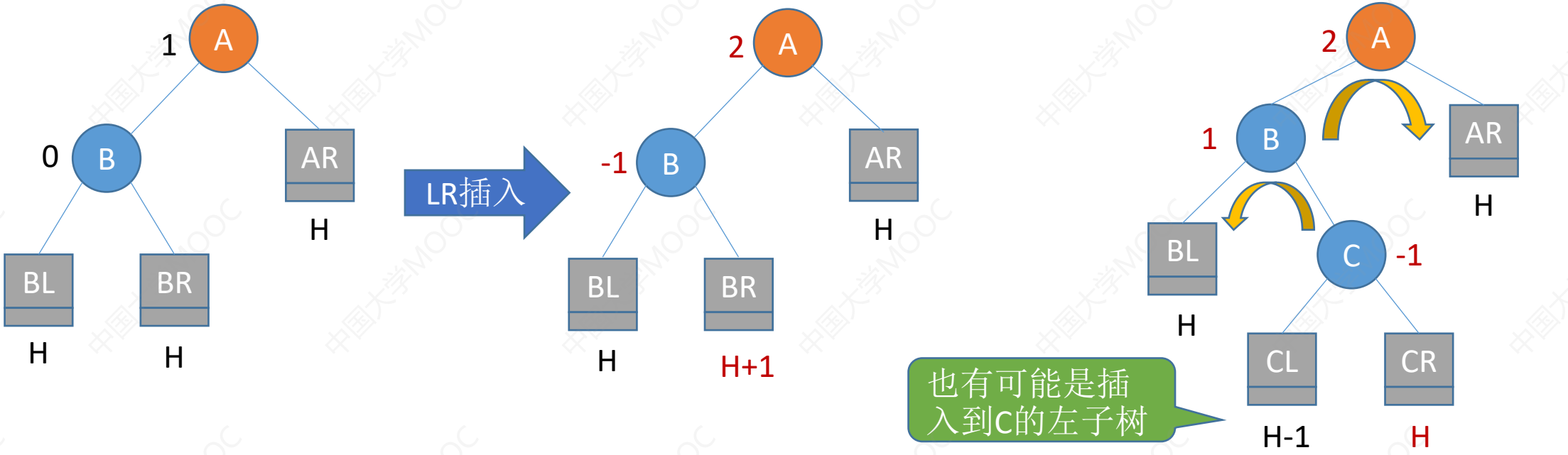
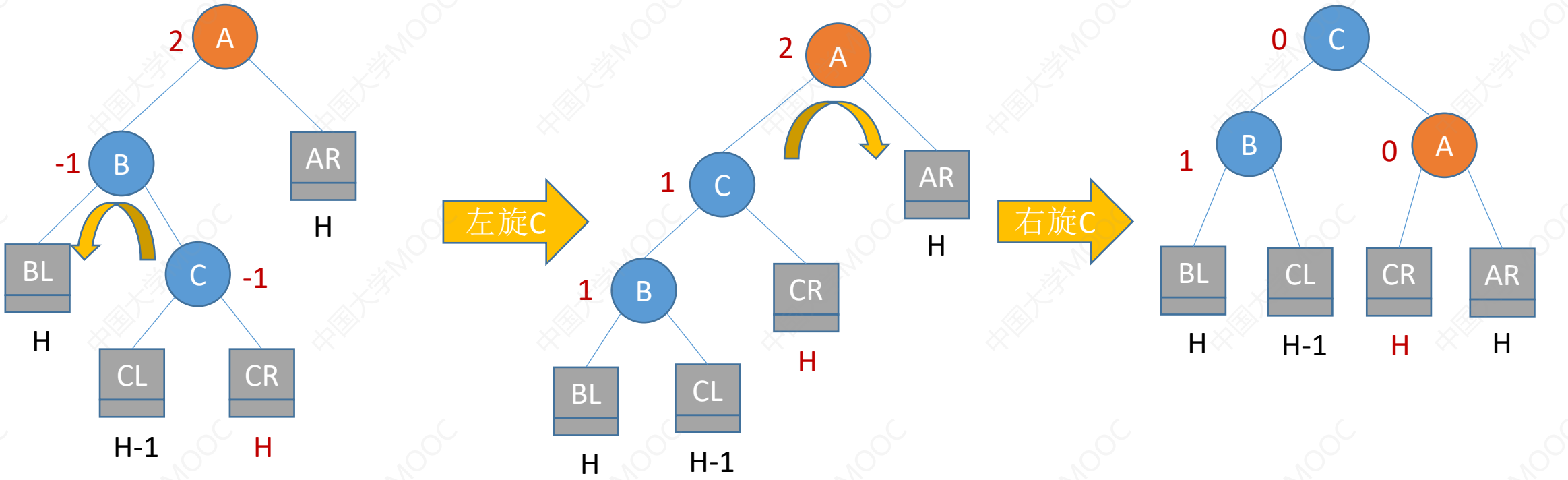
AL<A<BL<B<BR

调整最小不平衡子树（LR）



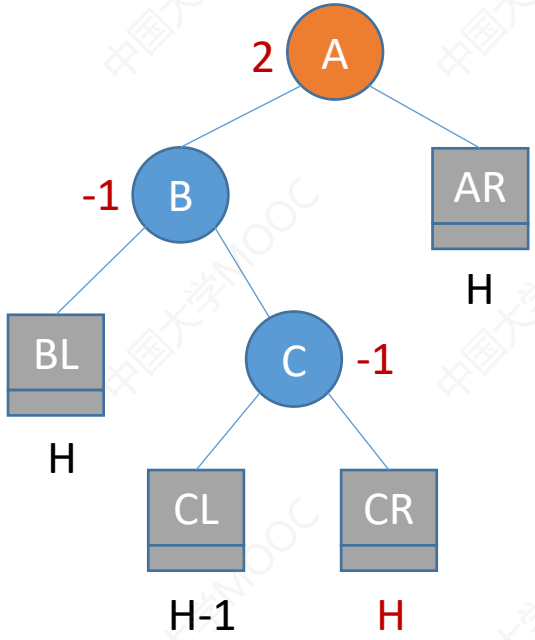
3）LR平衡旋转（先左后右双旋转）。由于在*A*的左孩子（L）的右子树（R）上插入新结点，*A*的平衡因子由1增至2，导致以*A*为根的子树失去平衡，需要进行两次旋转操作，先左旋转后右旋转。先将*A*结点的左孩子*B*的右子树的根结点*C*向左上旋转提升到*B*结点的位置，然后再把该*C*结点向右上旋转提升到*A*结点的位置

调整最小不平衡子树（LR）

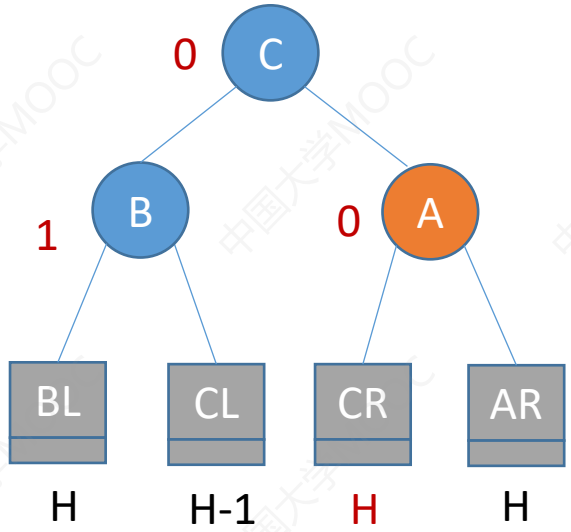


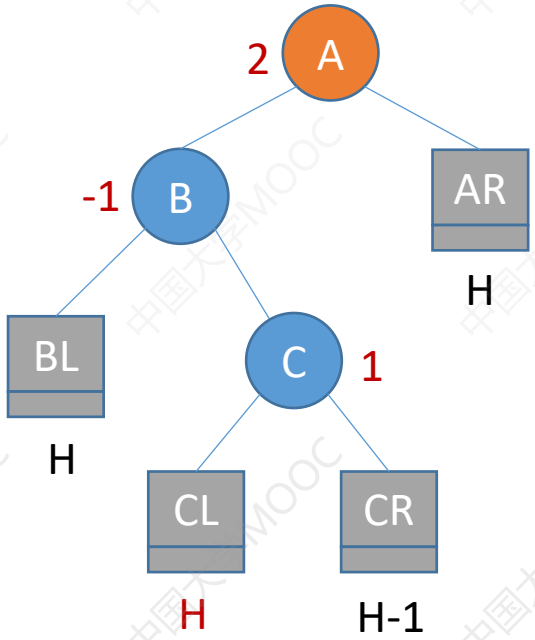
BL<B<CL<C< CR<A<AR

调整最小不平衡子树（LR）

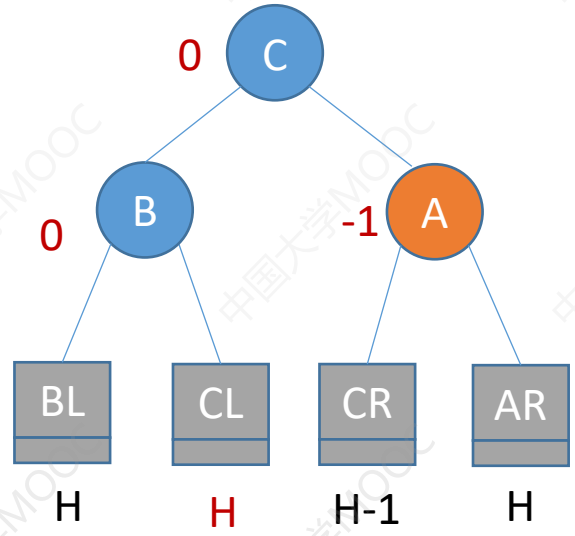


左旋C+右旋C



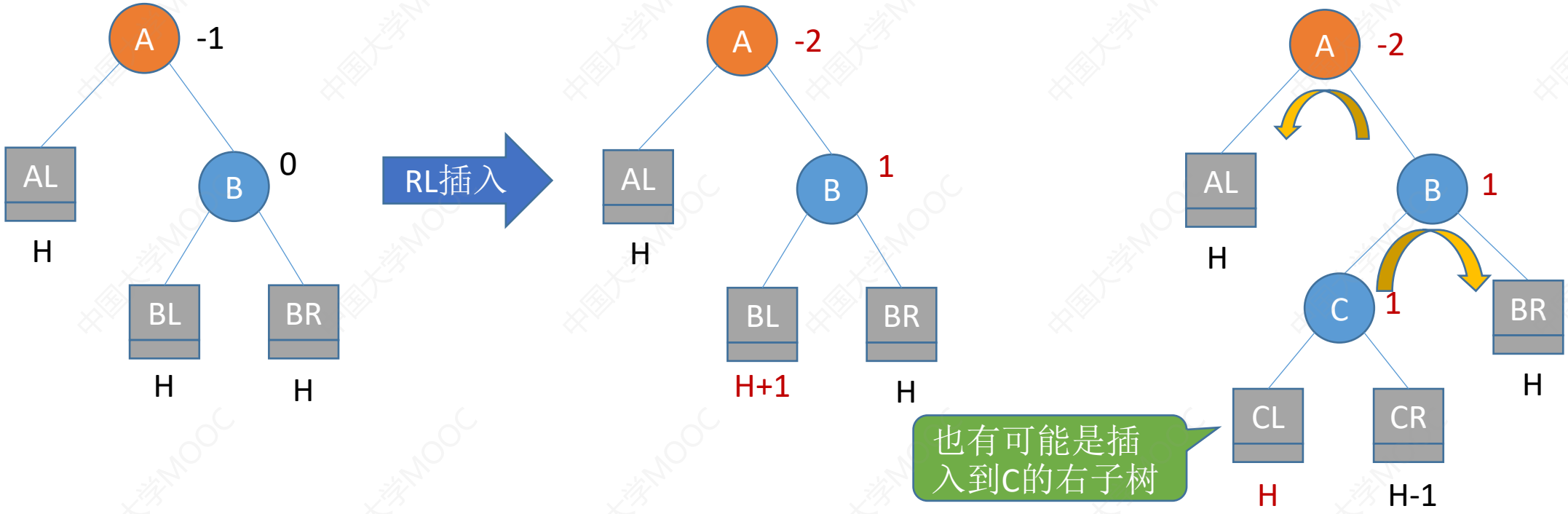


左旋C+右旋C



调整最小不平衡子树（RL）

AL<A<CL<C<CR<B<BR



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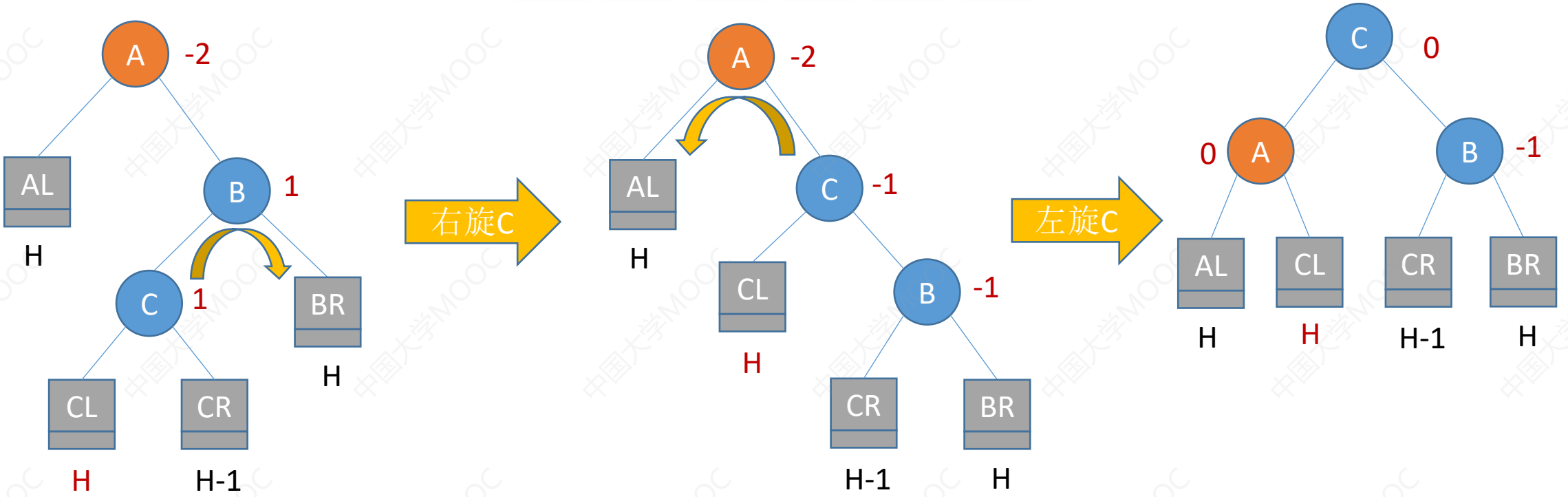
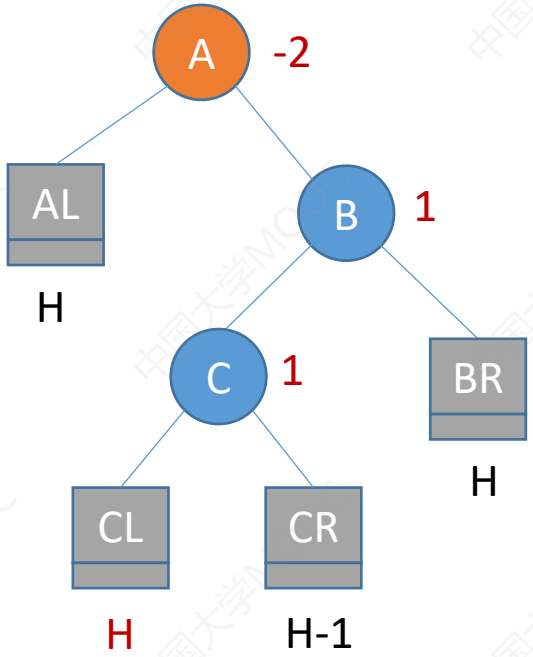
调整最小不平衡子树（RL）

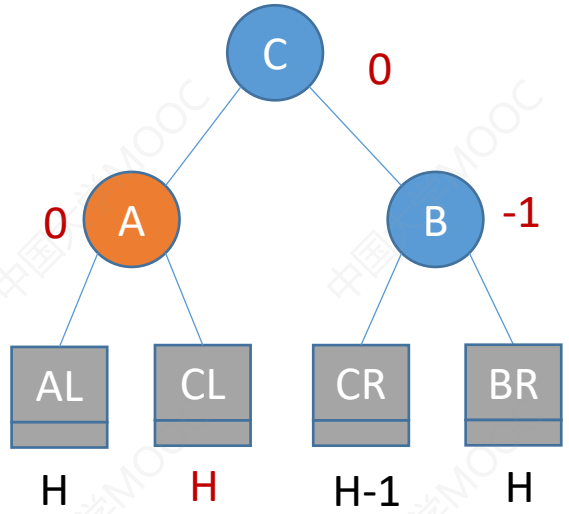


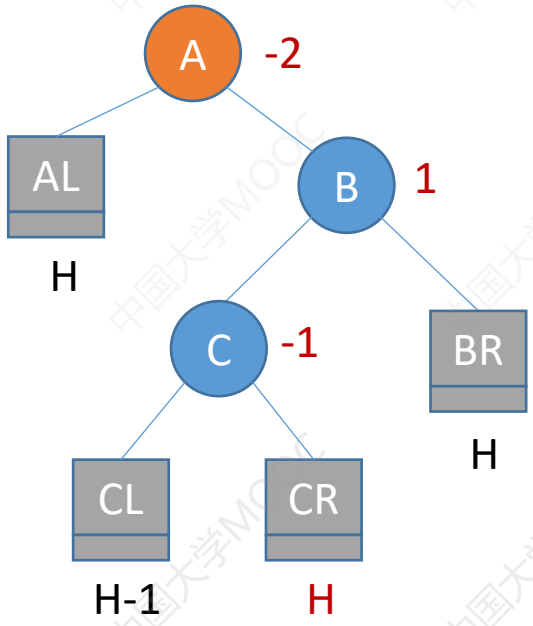
AL<A<CL<C<CR<B<BR

调整最小不平衡子树（RL）

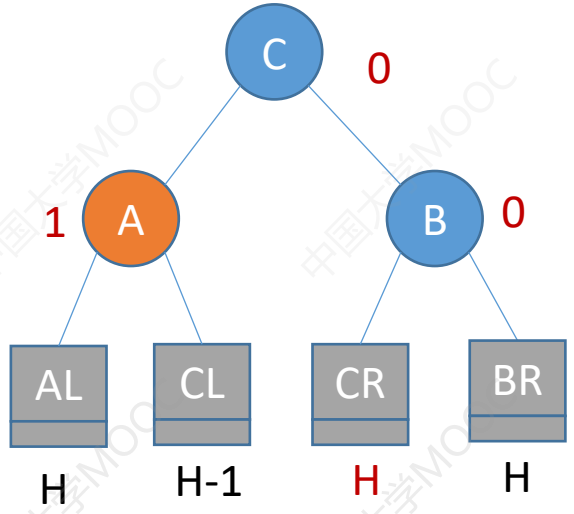


右旋C+左旋C





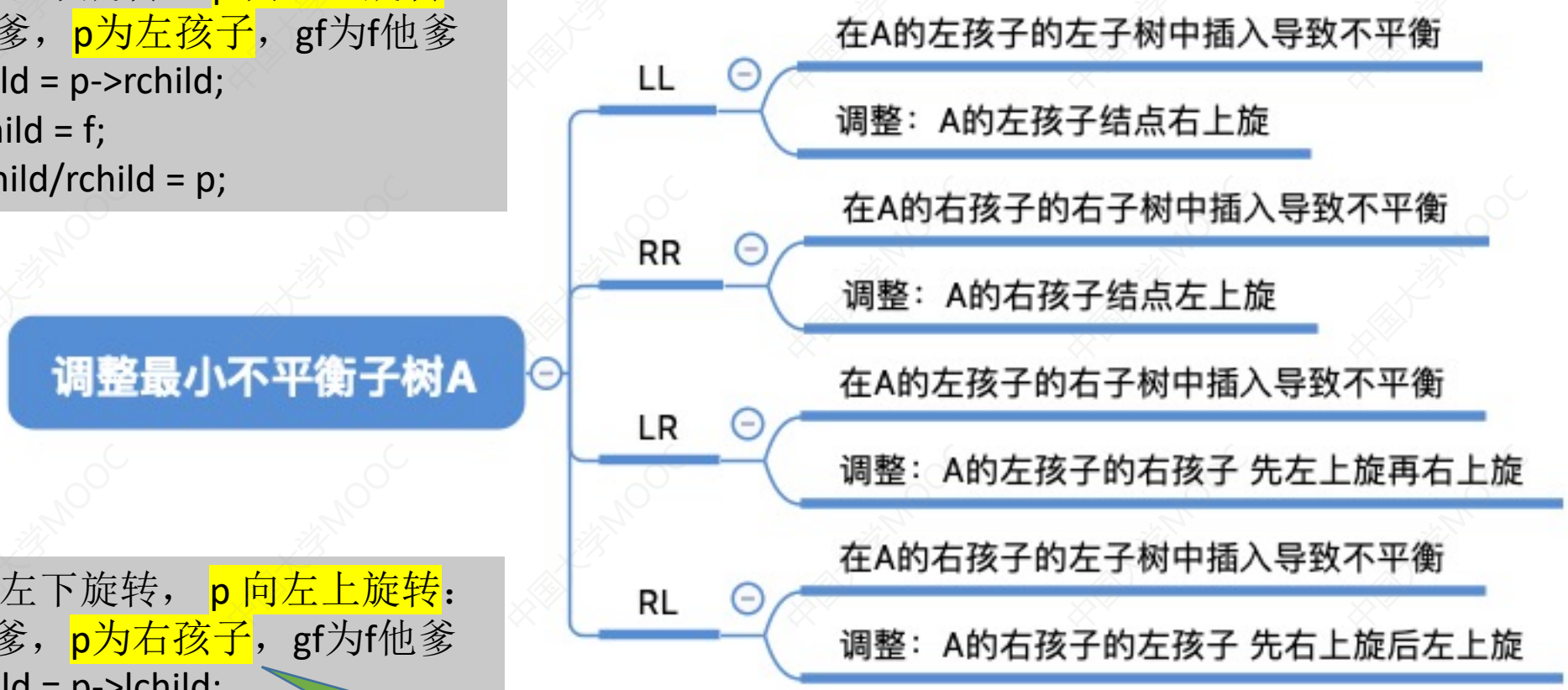
右旋C+左旋C



调整最小不平衡子树

只有左孩子
才能右上旋

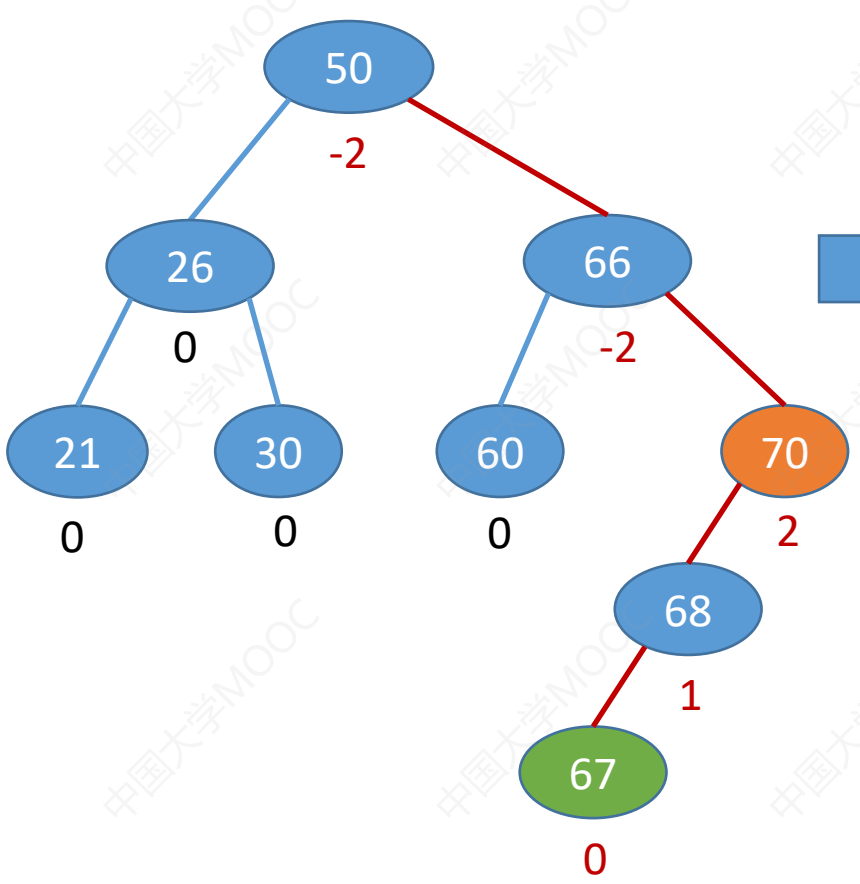
实现 f 向右下旋转， p 向右上旋转：
其中 f是爹， p为左孩子， gf为f他爹
① f->lchild = p->rchild;
② p->rchild = f;
③ gf->lchild/rchild = p;



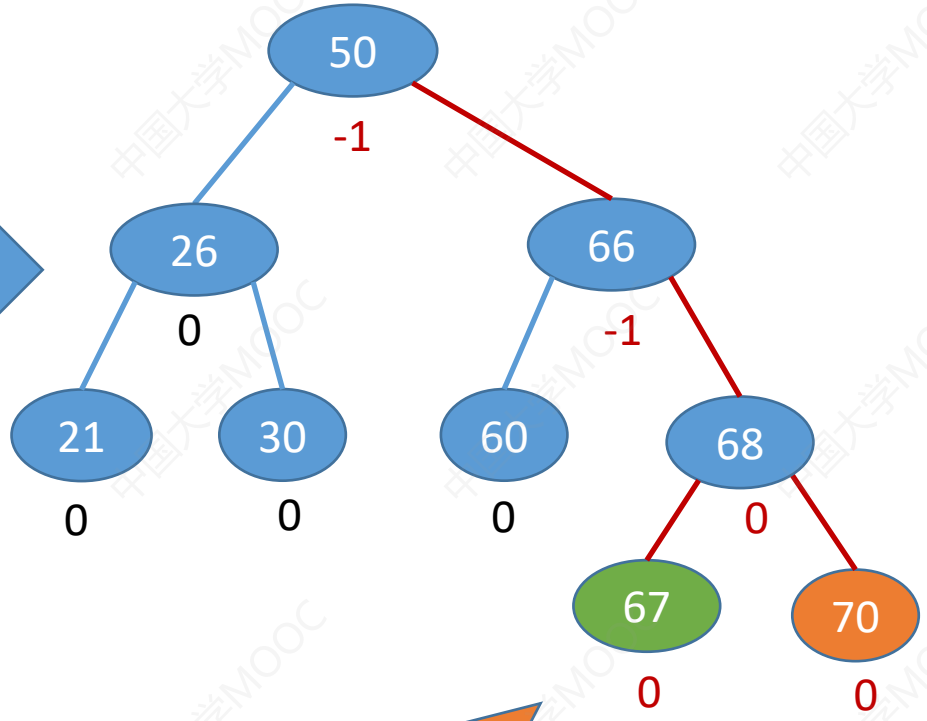
实现 f 向左下旋转， p 向左上旋转：
其中 f是爹， p为右孩子， gf为f他爹
① f->rchild = p->lchild;
② p->lchild = f;
③ gf->lchild/rchild = p;

只有右孩子
才能左上旋

填个坑



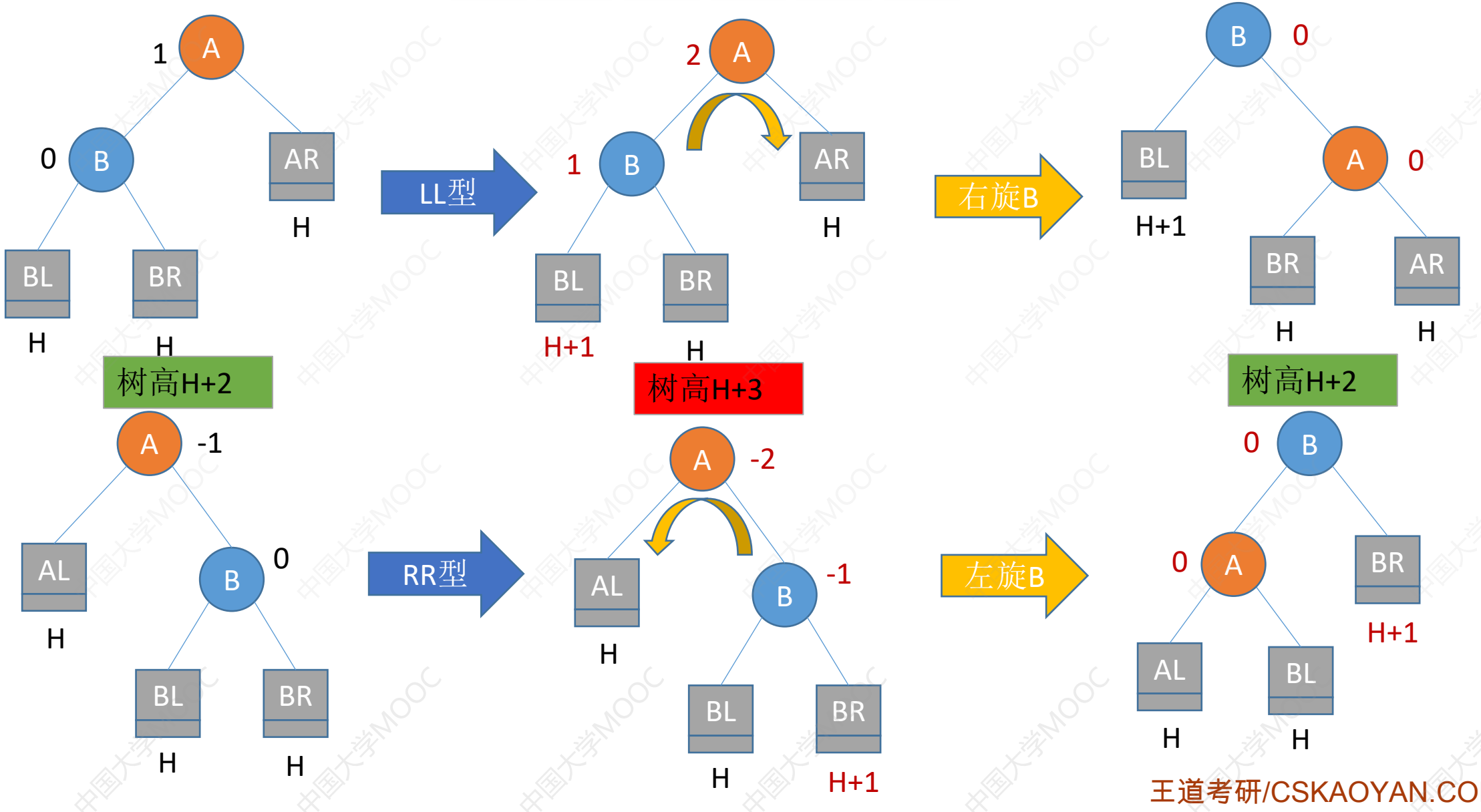
LL型，左孩子右旋



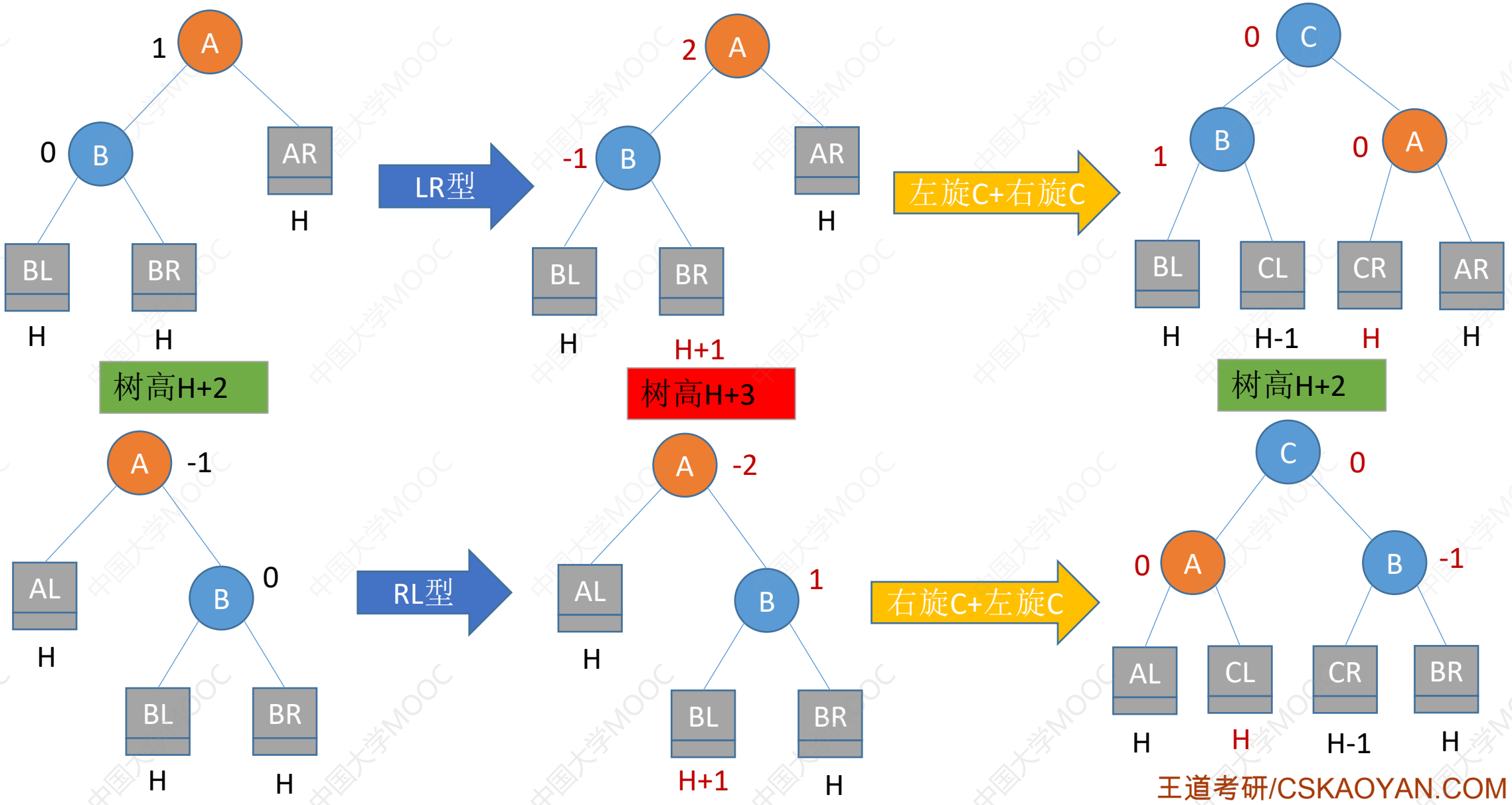
每次调整的对象都是“最小不平衡子树”

在插入操作中，只要将最小不平衡子树调整平衡，则其他祖先结点都会恢复平衡

填个坑



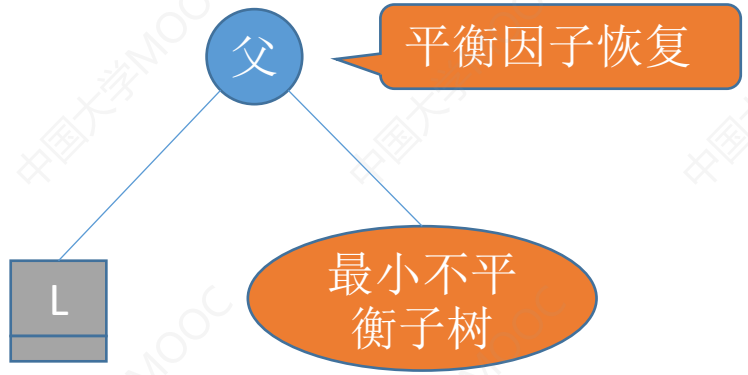
填个坑



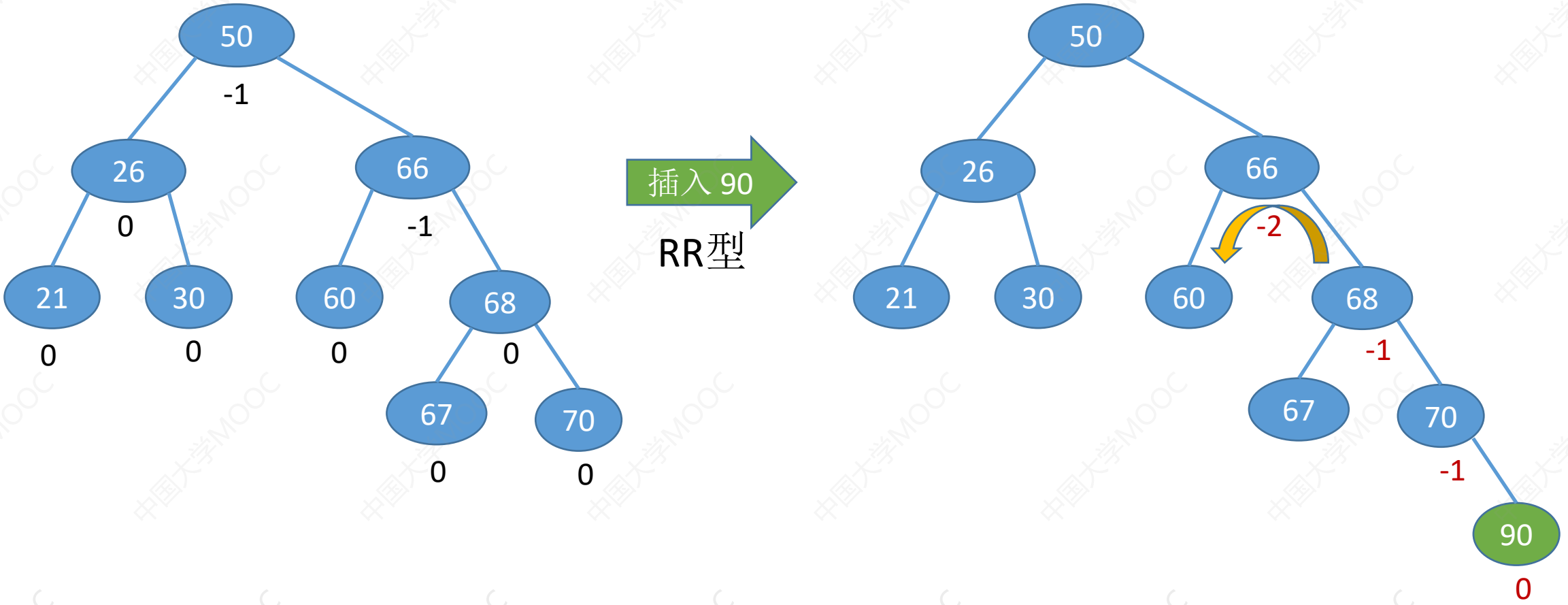
填个坑

插入操作导致“最小不平衡子树”高度+1，经过调整后高度恢复

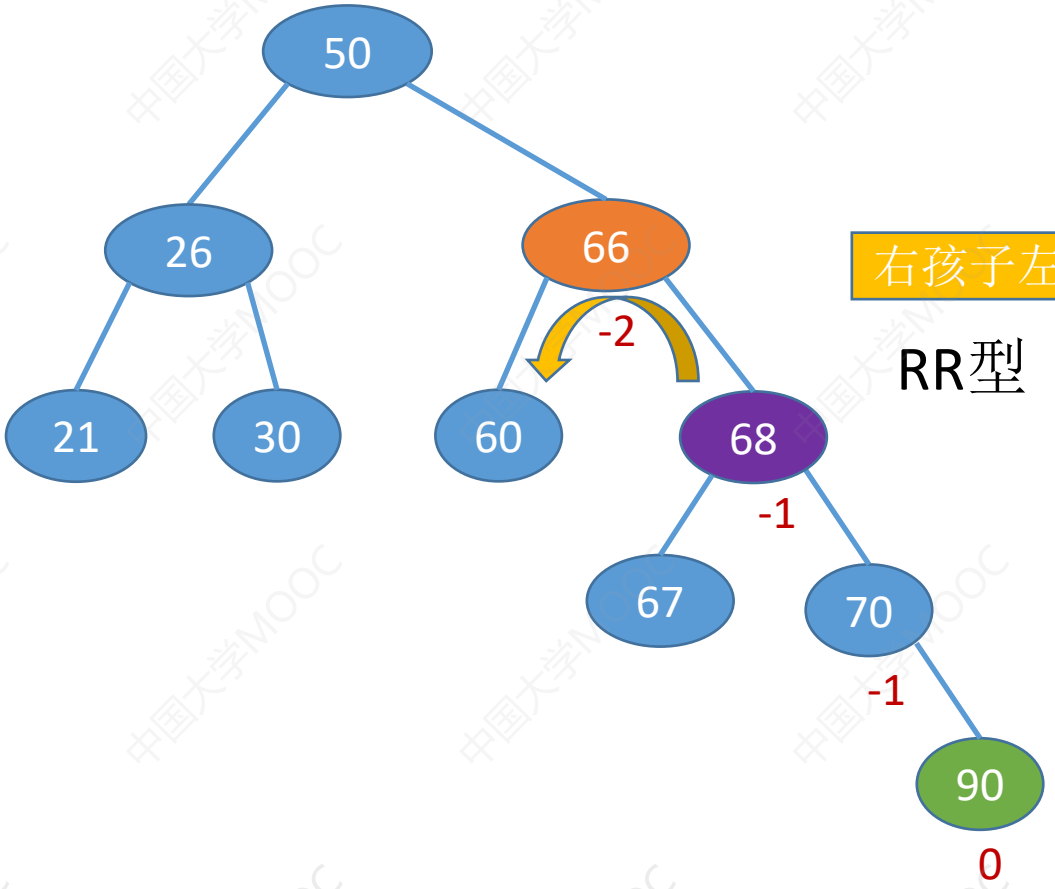
在插入操作中，只要将最小不平衡子树调整平衡，则其他祖先结点都会恢复平衡



练习

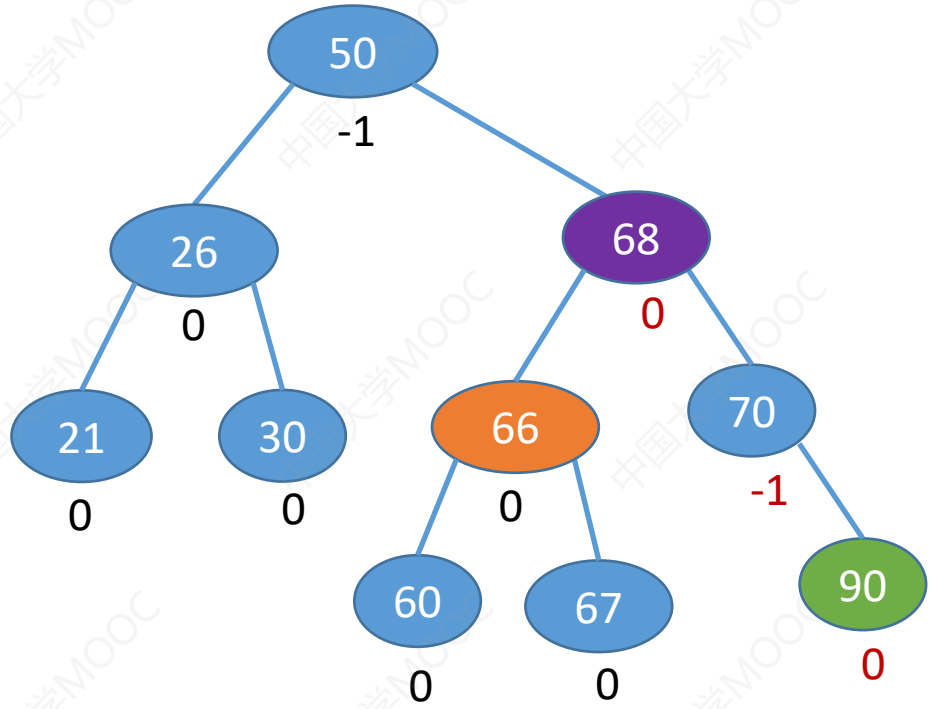


练习



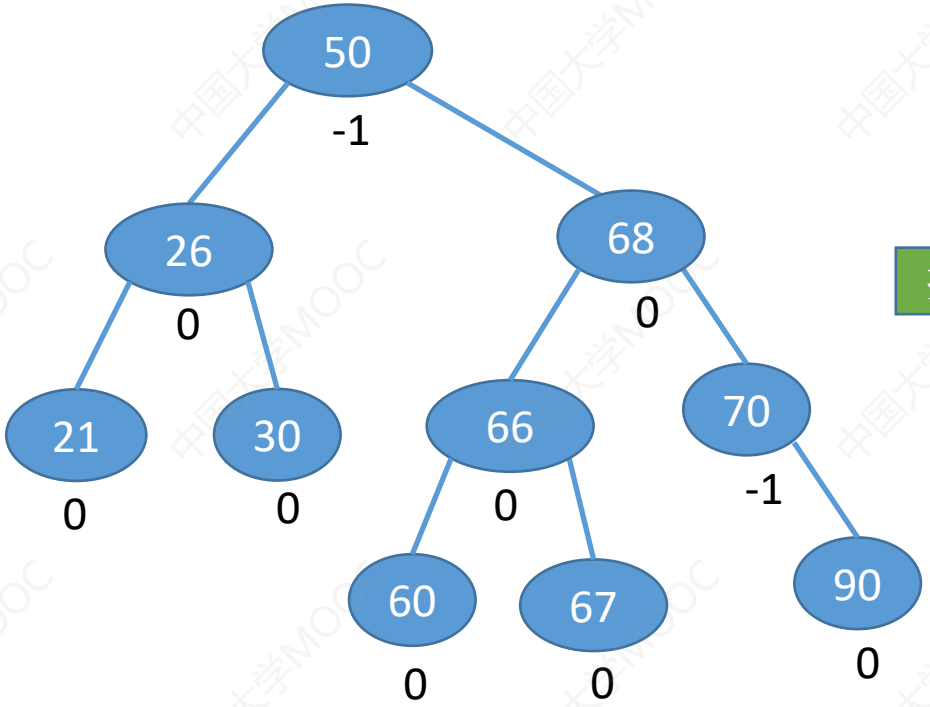
右孩子左旋

RR型



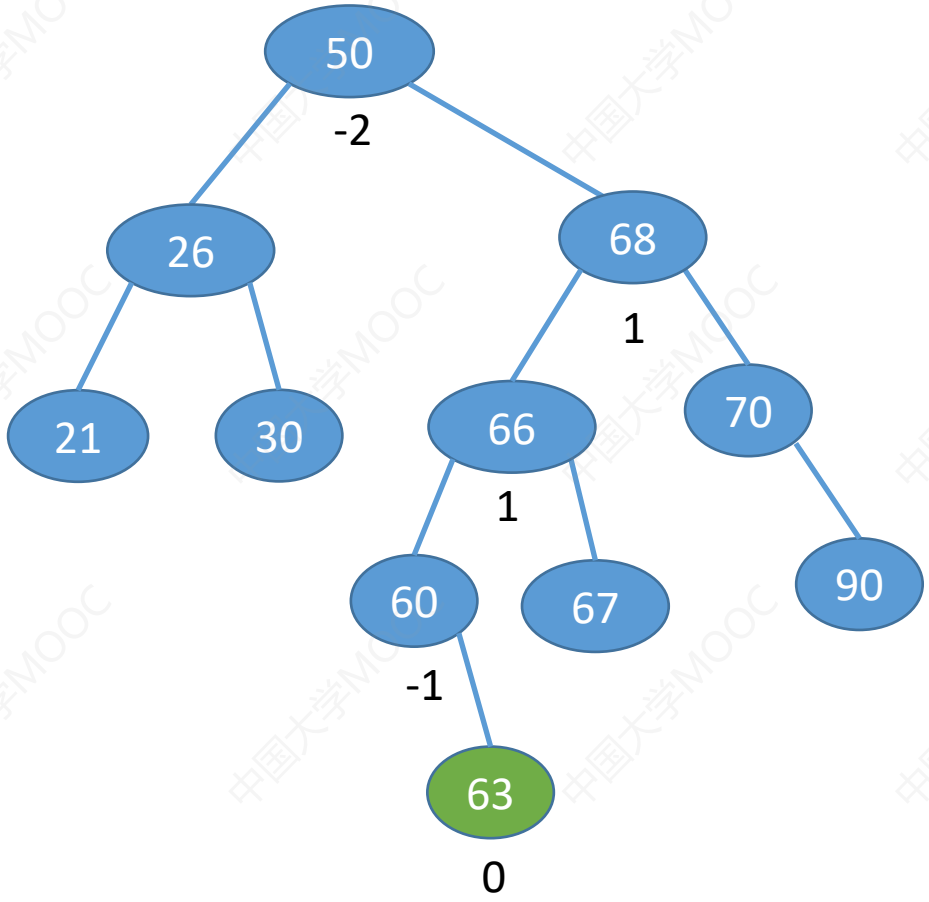
注意检查：是否符合 左<根<右

练习

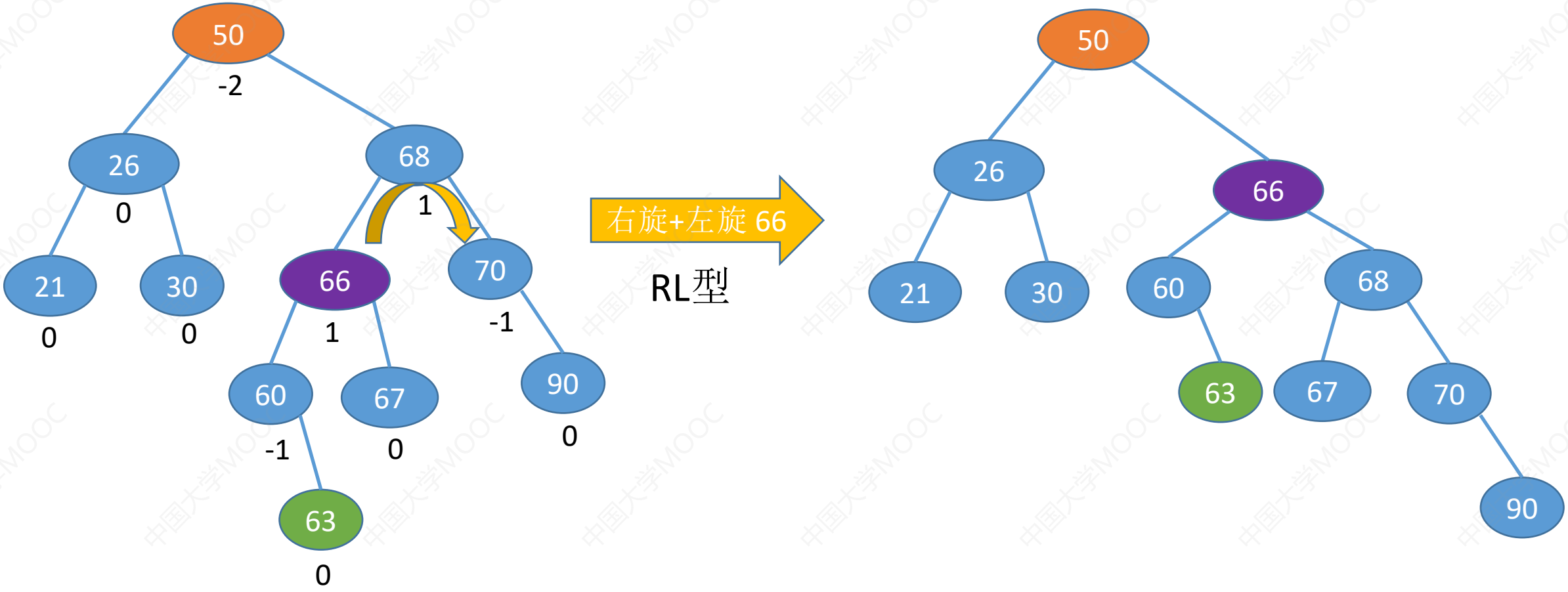


插入 63

RL型



练习



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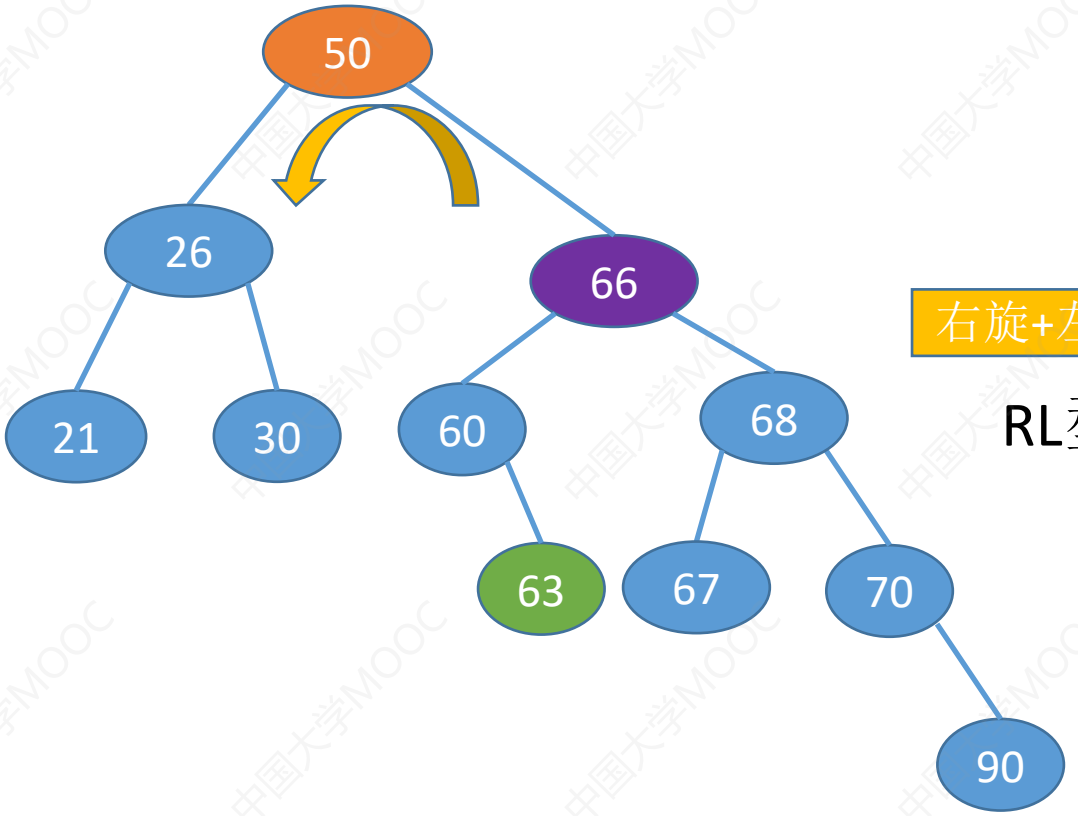
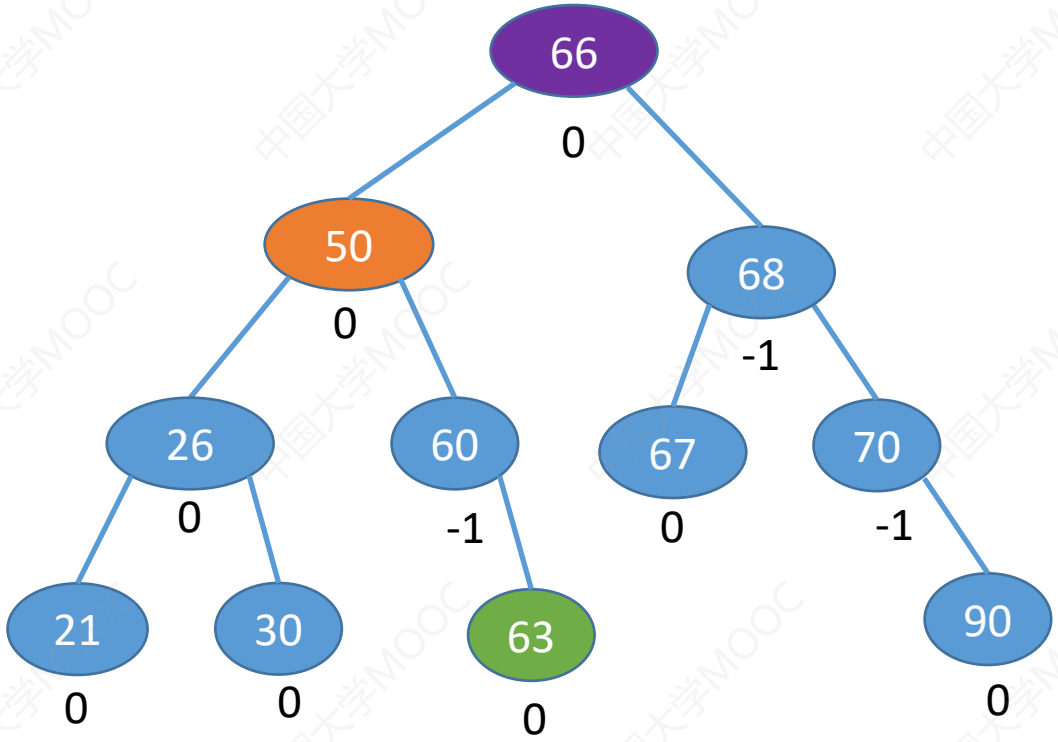


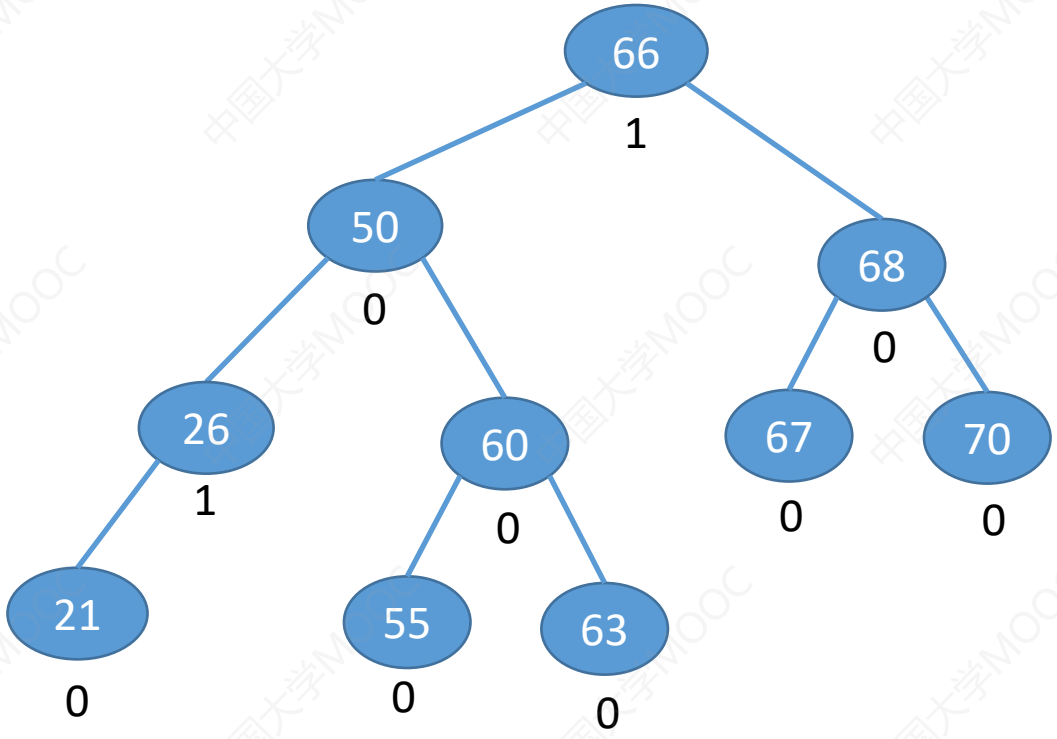
右旋+左旋 66

RL型



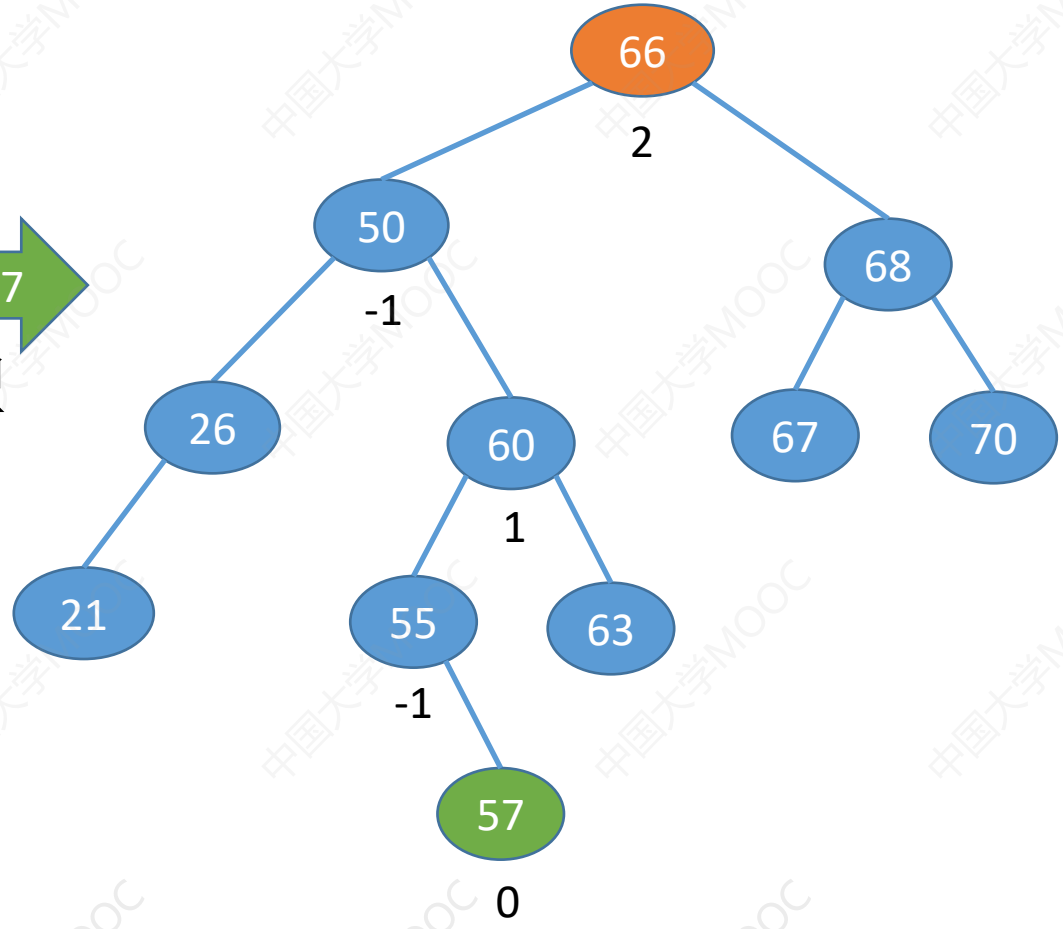
注意检查：是否符合 左<根<右

练习

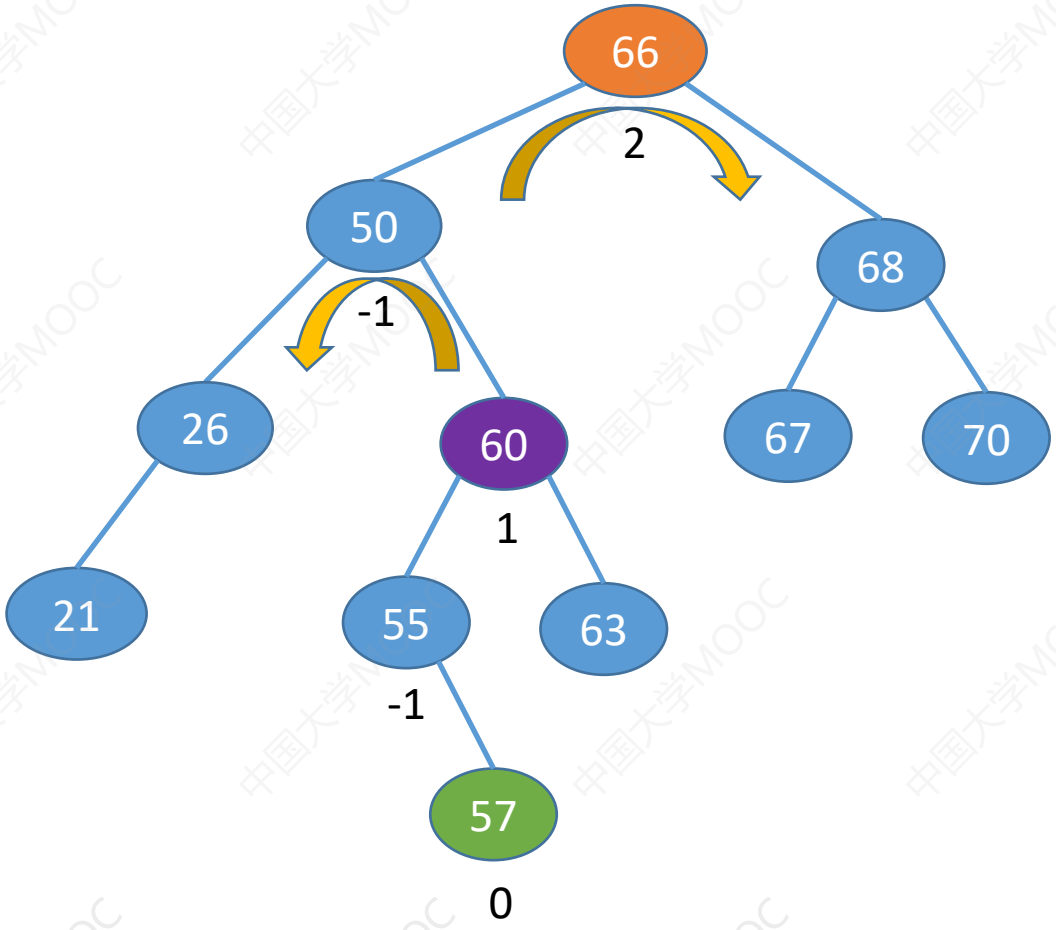


插入 57

LR型

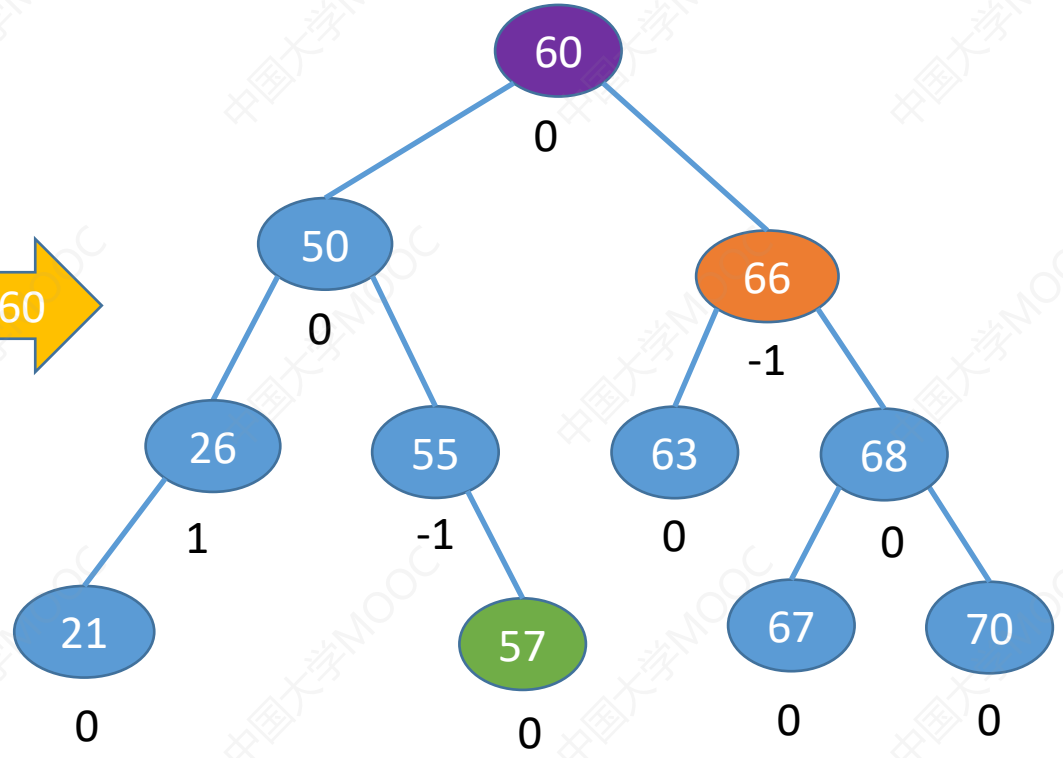


练习



左旋+右旋 60

LR型



查找效率分析

若树高为h，则最坏情况下，查找一个关键字最多需要对比 h 次，即查找操作的时间复杂度不可能超过 O(h)

平衡二叉树——树上任一结点的左子树和右子树的高度之差不超过1。

假设以n_h表示深度为h的平衡树中含有的最少结点数。

则有$n_0 = 0, n_1 = 1, n_2 = 2$，并且有$n_h = n_{h-1} + n_{h-2} + 1$

可以证明含有n个结点的平衡二叉树的最大深度为$O(\log_2 n)$，平衡二叉树的平均查找长度为$O(\log_2 n)$

查找效率分析

《An algorithm for the organization of information》——G.M. Adelson-Velsky 和 E.M. Landis ,1962

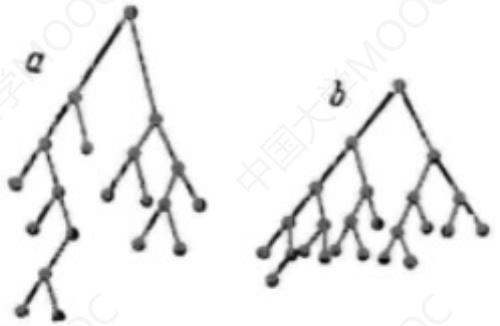


Figure 1

Figure 2

The recording algorithm is such that at each moment, the reference board is an admissible tree.

Lemma 1. *Let the number of cells of the admissible tree be equal to N. Then the maximum length of the branch is not greater than* $(3/2)\log_2(N+1)$.

Proof. Let us denote by N_n the minimum number of cells in the admissible tree when the given maximum length of the branch is n. Then it can be easily proven (see Figure 2) that $N_n = N_{n-1} + N_{n-2} + 1$.

When we solve this equation in finite remainders, we get

$$N_n = \left(1+\frac{2}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(1-\frac{2}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n - 1.$$

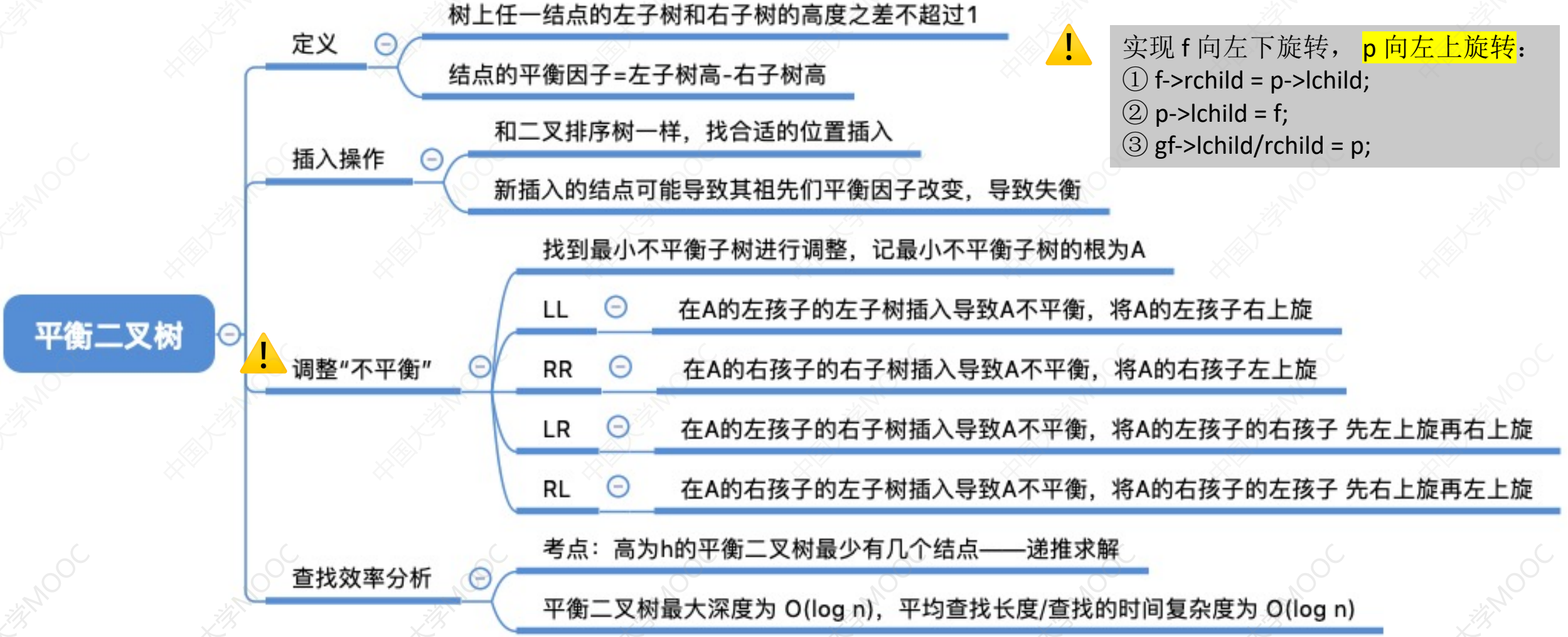
Whence

$$n < \log_{\frac{1+\sqrt{5}}{2}}(N+1) < \frac{3}{2}\log_2(N+1),$$

知识回顾与重要考点

实现 f 向右下旋转， p 向右上旋转：
其中 f是爹， p为左孩子， gf为f他爹
① f->lchild = p->rchild;
② p->rchild = f;
③ gf->lchild/rchild = p;

实现 f 向左下旋转， p 向左上旋转：
① f->rchild = p->lchild;
② p->lchild = f;
③ gf->lchild/rchild = p;



欢迎大家对本节视频进行评价~

学员评分：7.3.2_1 平衡二叉树

扫一扫二维码打开或分享给好友

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